

PROBABILITY OF ERROR

Transmit several symbols.

The probability of Error is an important metric to characterize the performance of a digital communication system.

Aim: To minimize prob of Error
 $\sim 10^{-6} - 10^{-8}$

Typical values of Prob of Error

For a given scheme, How to characterize the probability of Error?

After filtering & sampling

$$r(T) = a_0 \int_{-\infty}^{\infty} p(T-z)h(z)dz + \tilde{n}$$

Gaussian

To maximize SNR
 $p(T-z) = h(z)$

✓ Termed as Matched Filter

$$r(T) = a_0 \cdot \int_{-\infty}^{\infty} p^2(T-\tau) d\tau + \tilde{n}$$

$$= \int_{-\infty}^{\infty} p^2(\tau) d\tau = E_p.$$

Energy of
pulse shaping
Filter

$$= a_0 E_p + \tilde{n}$$

Transmitted
Symbol

Gaussian
Noise.

Pulse Energy

$$E\{\tilde{n}\} = 0 \quad \text{Zero mean}$$

$$E\{\tilde{n}^2\} = \frac{\eta_0}{2} \int_{-\infty}^{\infty} |h(\tau)|^2 d\tau$$

$$= \frac{\eta_0}{2} \int_{-\infty}^{\infty} p^2(\tau) d\tau$$

$$E\{\tilde{n}^2\} = \frac{\eta_0}{2} E_p.$$

$$\tilde{n} \sim \mathcal{N}\left(0, \frac{\eta_0}{2} E_p\right)$$

Noise after sampling
 Gaussian
 mean
 variance

$$\mathcal{N}(\mu, \sigma^2)$$

Gaussian of
 mean = μ
 var = σ^2

$$\mathcal{N}(0, 1)$$

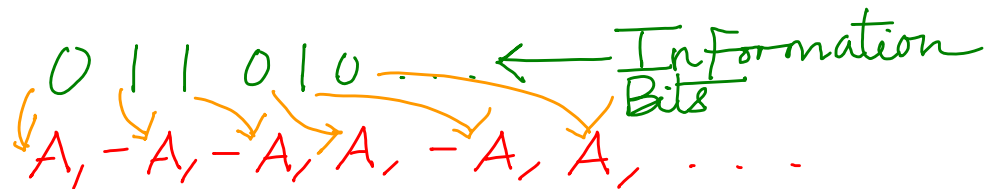
variance unity
 mean zero

Standard Normal.
Standard Gaussian

$$r(T) = a_0 E_p + \tilde{n}$$

Symbols must belong
 to a digital constellation
 in a Digital Comm
 System.

$$a_0 = -A \text{ or } +A$$



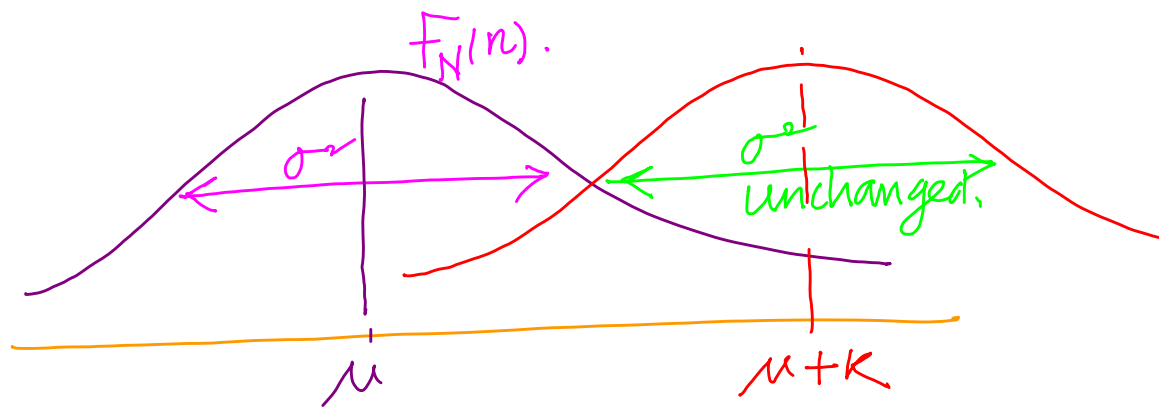
$$\begin{aligned} 0 &\rightarrow A \\ 1 &\rightarrow -A \end{aligned}$$

$$r(T) = \pm A e_p + \tilde{n}$$

$$\begin{cases} r(T) = A e_p + \tilde{n}, & a_0 = A \\ r(T) = -A e_p + \tilde{n}, & a_0 = -A \end{cases}$$

output after sampling.

Property: If $n \sim \mathcal{N}(\mu, \sigma^2)$
 Then $n + K \sim \mathcal{N}(\mu + K, \sigma^2)$
 constant $\rightarrow$ variance is unchanged.



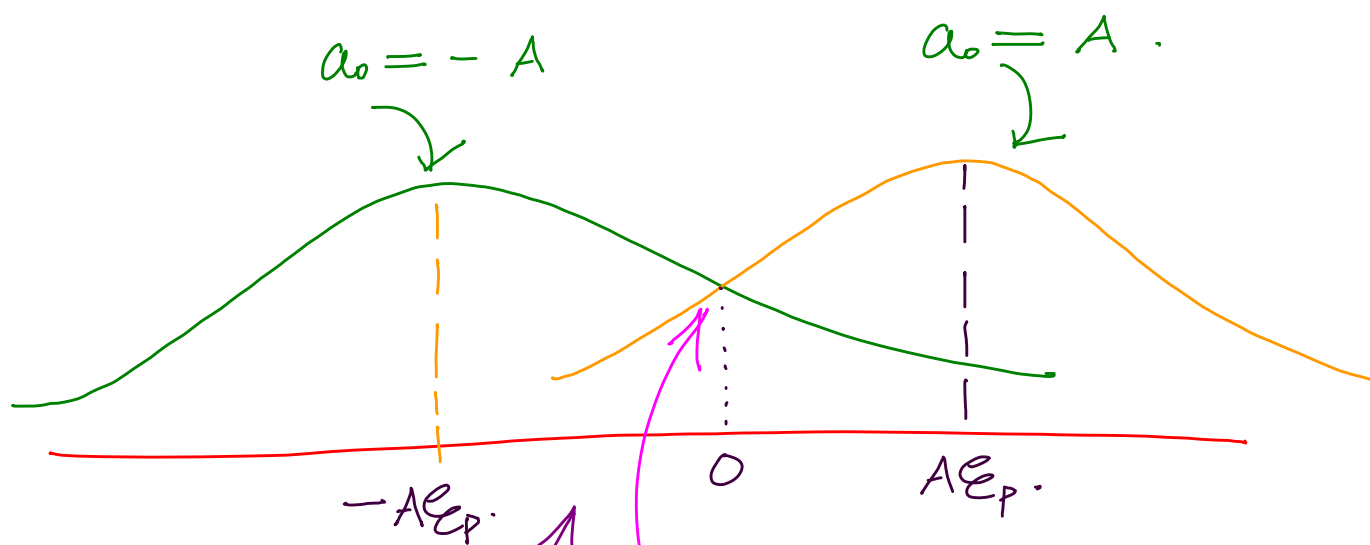
$$\sigma^2 = \frac{\eta_0}{2} \mathcal{E}_p$$

$$\Gamma(T) = \begin{cases} A\mathcal{E}_p + \tilde{\eta} \sim \mathcal{N}(A\mathcal{E}_p, \sigma^2) \\ -A\mathcal{E}_p + \tilde{\eta} \sim \mathcal{N}(-A\mathcal{E}_p, \sigma^2) \end{cases}$$

$\Gamma(T)$ is Gaussian
mean = $A\mathcal{E}_p$

$\Gamma(T)$ is Gaussian
mean = $-A\mathcal{E}_p$

Both cases variance is same = $\sigma^2 = \frac{\eta_0}{2} \mathcal{E}_p$



Point of intersection
 $= 0$
 By symmetry.

Both have same
 variance $\sigma^2 = \frac{\eta_0}{2} E_p$.